THE MATHEMATICS TEACHER

Volume XXXVI Number 8

Edited by William David Reeve

Mathematics for Electricity and Radio*

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WHEN I was invited to appear before this Society, I accepted with pleasure, but more or less on the spur of the moment. Later, upon giving the matter more serious consideration, I realized that I had taken on more of an assignment than was apparent at first glance. The reasons for this are simple; here I am appearing before a distinguished group of mathematicians and educators and I profess to be neither a professional mathematician nor an educator in the sense that the name implies. Furthermore, a schedule of sixteen hours per day allows little time to prepare a polished paper. Therefore, this morning I do not propose to expound any new mathematical theories or burden you with my ideas of the proper methods to be used in educating your students. The latter is your job and I feel that, in general, it is being accomplished in a splendid manner.

This does not mean, however, that we do not share common interests and common problems because every teacher in this room is engaged, either directly or by rendering constructive assistance, in training young people for their part in the war effort.

* Paper read at the Mathematics-Science Panel of the New York Society for the Experimental Study of Education, Hotel Astor, March 20, 1943.

As much as a crowded schedule has allowed, I have observed with keen interest the efforts of the teachers in our schools and colleges in attempting to assist the various training programs of the armed forces and industry. Unfortunately, you have encountered divergent policies from time to time and this has not made your task less difficult. Such differences of opinion are only natural when it becomes necessary to train people in such widely separated and specialized fields; and in such a short time. In spite of all this, when you consider the magnitude of the effort, the progress of the entire training program is truly amazing.

Since I have no official connection with shaping the educational policies of the Navy, there is no need for me to comment on your courses already in existence or those to be established in the near future. However, from conversations with members of your group it appears that while you are well informed on your phases of the program, you have been given little opportunity of becoming familiar with the training methods your students will encounter when they leave your schools. This is rather an unfortunate situation because in many instances, if you as a mathematics teacher, for example, knew just how your students were to be given addi-

tional training, you would be in a better position to assist them toward this end.

Because radio and communications play such a vital role in the war and require a tremendous number of men, both in the armed forces and in industry, it occurred to me that this audience might be interested in how large groups of radio students are being trained in various Navy schools and in certain civilian schools under contract to the Navy. Any description of the schedules, courses, and methods used in the school to which I am attached is applicable to practically the entire primary training program for navy radio technicians, because all schools engaged in this particular program adhere to a prescribed curriculum.

First of all, perhaps it would be well for me to explain what is meant by the term "radio technician." The radio technician should not be confused with the radio operator who is primarily concerned with operating radio and associated equipment in order to expedite communications. The radio technician's job is to maintain radio equipment in first class operating condition and, failing this for any number of reasons, to make the proper repairs and get the equipment back on the air with the least possible delay. The term "radio mechanic" is not applicable to our radio technicians because the latter's ability and knowledge of the field must be far greater than that which has become more or less accepted for the former. I like to think of our radio technicians as practical radio engineers and their performance in the fleet makes them worthy of the title and, at the same time, gives those of us who have had a part in their training, a feeling of deep satisfaction.

Everyone is aware of the existing shortages of critical materials, but none of them is more critical than time. Every hour this war is prolonged means acute suffering and untold hardships for great numbers of people. It is essential, therefore, that every man in the services be trained as quickly and thoroughly as possible in order that his services may be used on the fighting fronts. During his training period, the radio technician is of no use on the fighting front and it is imperative that he be trained in the shortest time possible, consistent with his ability to master the knowledge of his specialty.

The job of training a radio technician is far more difficult in these times than it was even a short time ago. The development of modern apparatus and new techniques has resulted in complex circuits; the understanding of which requires considerably more knowledge than is necessary for the servicing of an ordinary piece of radio equipment—for example, the typical broadcast receiver. Added to this is the time element previously mentioned—many men, capable of becoming good radio technicians under ordinary circumstances, must be eliminated as such because they are incapable of absorbing the training in the time allotted. This means that we should have a very high type of man as an applicant in order that time will not be wasted by a high rate of attrition.

This brings us to the consideration of the characteristics of men best fitted for this type of training. With regard to education, the present curriculum demands a facile mind to cope with the condensed teaching methods now in use. A background in high school mathematics is desirable, but failure to show such *credits* does not prevent consideration. While I mention the desirability of a good foundation in mathematics as the first requisite, this does not mean that we desire mathematical training to the exclusion of other important skills.

We would much prefer to work with a young man with four years of technical high school training or its equivalent, who exhibits a high level of native intelligence; that is, one who is "quick to catch on." By this I mean that men with purely an academic background and no technical aspirations or leanings are not the best material for our purposes.

Mechanical ability is a very important consideration, a considerable amount of which is necessary for successful comple-

tion of the course. It is evidenced by the candidate having an interest or inquisitiveness about machines, radio, or mechanical things in general. It is generally demonstrated by the men having built radio kits, model airplanes, or having been a general experimenter. Proficiency in any particular field is not required, but rather an interest in things in general.

Naturally, we would like all candidates to have a background of radio experience, but most of these men are already in the service or engaged in essential tasks in industry. However, this is not fatal to the program for we are continuing to turn out good technicians whose radio technique, prior to entrance in the service, consisted of playing a broadcast receiver. Last, but by no means least, some thought must be given to the candidate's personality. He must be capable of working under load conditions and thinking straight in tight spots; that is, he must indicate good reasoning powers under all conditions. Equally important is the fact that he must be acceptable to his shipmates because naval warfare has always meant that for weeks, and even months, men live and work together in crowded spaces without the stimulating influence of getting away from their surroundings for even very short periods.

To summarize, then, the candidate for radio technician training must first indicate an interest is this type of work. Secondly, he should be a highly intelligent individual, with mechanical aptitude, and a good background in mathematics. In addition, he must be able to get along with others and be physically capable and willing to work a fifteen-hour day, during his training, and like it. At times, upon the completion of his training, he will encounter longer hours.

With regard to age, our experience has shown the younger man to be a better risk independent of his original ability. While applicants of all ages are analyzed, the attrition is too high for the group over forty years of age.

Having outlined the desirable characteristics of our candidates, let us see what we are actually getting. The average student, upon entering our school, is twenty-five years of age, has had twelve and one-half years of formal schooling, has an Otis higher examination percentile of 75.8, and has demonstrated mechanical ability. This is an excellent product and if we continue to get this type of man, in sufficient quantities, we can meet the needs of the service for radio technicians.

The great majority of candidates for this training are selected while they are undergoing indoctrination at the several naval training stations. After screening by general classification tests and personal observation, the best qualified men are further screened with the Eddy test which is sent to Chicago for grading and analysis by a staff under the supervision of Commander W. C. Eddy. The successful candidates are then transferred to a pool awaiting further transfer to one of the schools engaged in primary training for radio technicians. This does not mean that a man's time is wasted in awaiting transfer to one of the schools. A course of instruction, which we call "Pre Radio," is given during this time, and consists of a mathematics review, the fundamentals of electricity, the use of hand tools, and naval indoctrination. The main purpose of this course of instruction is to get the man back into the routine of study so that no time will be wasted in doing this at one of the regular schools. This policy has produced very gratifying results.

The length of the entire course is 36 weeks and is designed to turn out a thoroughly competent radio technician; as previously mentioned, a man with a sound knowledge of radio, capable of maintaining and repairing a wide variety of radio apparatus. Because the civilian schools engage only in the primary training and send their graduates on to navy schools, I will discuss only this primary training which is fundamental in nature and prepares the student for advanced radio subjects and the study of specific equipment. The length of the primary course is twelve weeks.

The twelve-week course is divided into three units of four weeks each. All except one of the schools engaged in this program convene a new class every four weeks—at our school, we take on a new class every two weeks. When a new class is assembled, the majority of new students report for duty on Friday or Saturday for the class convening the following Monday. This allows time for barracks assignment, unpacking and becoming acquainted with their new station.

The first day of school is spent in registration, entrance examinations, acquainting the students with the physical layout of the school and the daily routine, a thorough physical check-up, issuance of school equipment, and the other numerous details connected with assembling a new class—with which you are all familiar.

Although, as previously mentioned, our students are carefully screened before reporting to our school, we continue to give entrance examinations which, when properly analyzed, serve as an additional check on the effectiveness of selection, give us a good picture of the class as a whole, and serve as a distinct aid later on in judging individual progress.

The entrance examinations include elementary electricity, mathematics, an Otis S A higher examination, and a mechanical aptitude test. As far as the mathematics is concerned, we are more interested in determining whether or not the student can just plain "figger" than we are in his ability to recite rules and proofs. In general, the results of the entrance examinations, when compared with those and progress marks of former students, indicate the type of work that may be expected during the course. Also, they point out immediately those students requiring additional help from the instructors.

On the second day (Tuesday) school really begins and the men start settling down to the normal school schedule. This daily schedule begins with reville at 6:00 A.M. when everyone arises and spends the next half-hour in washing, shaving, making his bunk, and cleaning his room. In this connection, at our school the barracks spaces are divided into rooms designed for six men. In addition to the usual bunk and clothing lockers, each room contains a large study table lighted by fluorescent lights. The breakfast hour begins at 6:45 and everyone is required to be in his assigned classroom for muster at 7:45. Four lecture periods follow, ending at 11:15. From this time until noon all students, divided into platoons, engage in military drill and physical exercise. From noon until 1:00 P.M. is taken up by dinner and relaxation.

The afternoon session, which lasts from 1:00 until 4:20, consists largely of laboratory work. From 4:20 until about 4:50 every student reports to his assigned space, and participates in a general cleaning of the school and barracks spaces.

Supper is served at 5:00 P.M. and from then until 7:30 the student's time is his own. Voluntary athletics are encouraged during this time, and consists of ball games between various classes, touch football, playing catch, and the numerous other ways in which young men blow off a little steam.

At 7:30 all students go to their rooms in the barracks and study under supervision until 10:30. At this time, anyone is at liberty to retire, but few do. The great majority continue their studies until 11:00 when all lights are extinguished and everyone must go to bed.

This schedule is maintained five days per week with one-half day of school on Saturday. No laboratory work is scheduled Saturday; the morning being devoted to reviews, discussion periods, or quizzes on the week's work. The latter part of the morning is devoted to giving the school a good cleaning or to inspection, whichever is directed by the Commanding Officer.

From the foregoing it might appear that our laboratories and classrooms are in use but one-half of the school day. This is not the case because each class is divided into two sections and the afternoons are divided into four lecture periods as in the

morning. For example, if section A attends lectures in the morning, section B will be engaged in the laboratories. That afternoon, section A is in laboratories while section B attends lectures.

So much for the daily routine—now for some details of the course.

During the first four weeks of the course, formal classroom instruction is confined to two main subjects; the fundamentals of electricity and applied mathematics; each subject being given equal time, that is, two hours per day per subject. In addition, some time is spent in demonstrating the use of tools but, in the main, this instruction is confined to the laboratory. The course in electricity begins with fundamental concepts and, in the first four weeks, is confined to direct currents; emphasis being placed on the action of the more common circuits encountered in actual practice, and measuring instruments used therewith. In fact, the material covered is contained in the electrical section of almost any good high school physics book, except in our course the theoretical considerations are reduced to a minimum and emphasis placed on the practical. Fortunately, this is a splendid course with which to key the instruction in mathematics. For example, about the first things to be considered are the electrical units with their conversion into kilounits, milliunits, microunits, megaunits, and so forth. Where better a place to introduce the powers of ten as an aid in computing and properly placing the elusive decimal point? More later on this.

By the time the student is ready to study series circuits, he has had time to review the early parts of algebra and has finished with simple equations; that is, equations containing but one term in numerator or denominator if a fraction is involved. In addition, he has had time to learn the operations of multiplication, division, squaring, and extracting square root on the slide rule. In general, this is all the mathematics he needs for the solution and analysis of simple series circuits.

The solution of parallel circuits is facilitated by the use of fractional equations and the mathematics instruction is arranged so that he studies these just before learning about parallel circuits.

In the fundamentals of electricity course there is a gap between the consideration of parallel circuits and the more complex circuits. This time is taken with the study of the magnetic circuit as an introduction to the operation of electrical equipment and electrical measuring instruments. Here the student must use the knowledge gained in the study of series and parallel circuits and thereby make daily practical applications of his knowledge of mathematics. During this gap the student reviews simultaneous linear equations and quadratic equations. He then makes immediate use of them by applications to circuits requiring analysis by Kirchhoff's laws. This completes the course of study for the first four weeks period. During the second four-weeks period daily instruction consists of one period of applied mathematics, one period of alternating current circuits and two periods on the fundamentals of radio. Again mathematics is carefully keyed to electrical circuits in that during this time the student must have enough of the essentials of plane trigonometry, simple plane vectors, and elementary vector algebra for the understanding of alternating current circuits. This is the most difficult month for quite a few of the students because many have not studied trigonometry in high school.

Mathematics, as such, is not taught in the third four-week period; this time being devoted to one hour of electrical machinery, one hour of communication circuits, which is really a continuation of alternating current circuits applied to radio, and two periods per day on radio circuits. However, all of their mathematical knowledge is applied in every subject.

While I have spoken briefly of the relation between instruction in applied mathematics, electricity, and radio, it might be well to consider the instruction in mathematics by itself. As previously stated, the

course in mathematics begins with the study of the powers of ten. To us, this is a very important subject because, unfortunately, the radio student and the radio engineer are required to handle cumbersome numbers, these numbers ranging from extremely small fractions of electrical units to very large numbers, as represented by radio frequencies. The fact that these wide limits of numbers are encountered in the same problem does not simplify matters; especially for the student who is rusty on his "figgering." This situation is becoming more complicated owing to the trend to the ultra-high radio frequencies with attendant smaller fractions of units represented by circuit components. For example, an inductance of 45 microhenrys connected to a capacitor of 250 micromicrofarads will resonate to 1500 kilocycles. Here we have an inductance expressed in millionths of the basic unit, the capacitance in millionths of millionths of the basic unit, and the frequency in terms of a thousand times the basic unit. When these values are converted to basic units, all in one formula, placing the decimal point becomes a major task for some of these boys. By expressing all numbers as a number between one and ten, times the proper power of ten, and using them in this form for computation, placing the decimal point presents little difficulty.

Perhaps you think that I place too much emphasis on the powers of ten as a working tool, but remember, our students are also taught the use of the slide rule and until someone comes out with a slipstick that furnishes decimal points, the powers of ten offer the most direct method of placing the decimal. In this connection, the important item of placing the decimal has not been sufficiently stressed by most writers of slide-rule instruction books. They resort to two dodges; that the decimal point may be fixed by inspection, or force the student to remember several rules. The first is impossible of accomplishment, and the second causes the student to lose interest. Accordingly, many beginners interested in using the slide rule have become discouraged and given it up as a hopeless task. We teach the use of the slide-rule simply because we do not want to waste our student's time in performing arithmetical computations with a pencil when the same work can be accomplished in a fraction of the time with the rule.

Now it must be admitted that when you proceed from the powers of ten, take algebra through quadratics, study some trigonometry and elementary vector algebra in two months, you have a rugged mathematics course. Naturally, this would be impossible of accomplishment except for several reasons; first, we have a very superior student body; second, their mathematics instruction is pointed toward a definite goal and every period they are shown where this mathematics is put to work; third, and probably the most important of all, the students are intensely interested in the subjects being taught; fourth, every instructor is a graduate of the school and the majority have spent years in radio work in the fleet, and they know exactly what the student will encounter when he leaves the school. Added to all this is the fact that the student is not studying unrelated subjects; he is there to learn radio.

Actually, we would like to teach no mathematics at all in order that the student's time could be spent in learning radio. However, the incoming students do not have sufficient mathematical ability and mathematics is essential as a tool in speeding up the learning process in radio and electrical subjects. We are actually saving time by taking time to refresh and teach the students mathematics and the use of the slide rule.

Although I realize that the majority of those present are interested in mathematics, I would like to point out that instruction is supplemented by laboratory work and visual aids, such as classroom demonstrations, slide films, and moving pictures. Our ultimate goal is for the student to spend as many hours in the laboratory as he does attending lectures, that is, 80 hours per month. In the primary schools,

because of lack of equipment and spaces, we have not quite attained this schedule but we are constantly striving for it. In this connection, we are trying for a maximum of two men on any one laboratory assignment.

With regard to visual aids, whenever possible during a lecture we demonstrate concepts with actual equipment. This not only speeds the learning process, but leaves a lasting impression on the student.

The Navy is engaged in a tremendous visual aids program and we now have large numbers of slide films and moving pictures on a wide variety of subjects; with many more to come.

Therefore, by supplying the student with the necessary mathematics, by delivering a series of well planned lectures on the many phases of radio, emphasing these lectures by demonstrations with actual equipment, showing slide films or movies which are carefully keyed into the course, and finally assigning laboratory work to demonstrate and further teach the principles, we believe that we have a well rounded course in the fundamentals of radio.

I know that almost all of you are extremely interested in the value of service training and how it can be fitted into civilian life after the war is won. Those of us engaged in the training of radio technicians have no worries in this connection because we *are* training men for jobs in civilian life. The good radio technician of today is the television engineer of tomorrow. He will operate the hundreds of new frequency modulated broadcasting stations. He will build and design the new receivers. Furthermore, he will play an active part in the broad field of electronics, which will produce undreamed of apparatus both for industry and everyday use.

I have attempted to emphasize the fact that we are continuing to receive a very good grade of students. This does not mean, however, that we are satisfied with them, if for no other reason that satisfaction would indicate perfection and this is not Utopia. Even if our incoming students were satisfactory, we would still be facing an undesirable situation because the percentage of those high school graduates selected is entirely too low. It is my belief that ninety per cent of the high school graduates should be able to successfully complete the radio technicians training in the time allotted; but this is impossible. The results of screening over fifty thousand high school graduates show three glaring deficiencies; that is, deficiencies as far as our field is concerned. First, they have received insufficient mathematical training; second, their knowledge of things scientific is meager; and third, which is very important, too many have absolutely no mechanical aptitude.

Let us briefly consider these three items. Our average student has had two and one-half years of high school mathematics, which generally means that one year of this has been spent on geometry. This should be fair preparation except that somewhere along the way, he has forgotten a lot of this mathematics and, more important, has lost the knack of accurate handling of figures; if he ever had it.

The lack of knowledge in physics is appalling but understandable when you consider graduates taken from the entire country.

Less understandable is the general lack of mechanical aptitude because this one thing is one of our most important heritages. This type of training has been neglected in too many schools.

However, I am not going to worry about the future of our schools and the manner in which our youth will be educated. This is a war of brains and machines and requires technicians to fight it. I am sure that when our fighting men return to peacetime pursuits, they will insist that their children be trained to take their places in a better world of expanded technical horizons.

Remedial Arithmetic in Senior High Schools

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IT IS scarcely necessary to remark that many pupils in high school are found who are deficient in the fundamentals of arithmetic. Some do not know the primary facts of addition, subtraction, multiplication, and division, to say nothing of such advanced processes as simple fractions.

Secondary schools once took the position that it was the duty of the elementary schools to teach the basic facts of arithmetic. However, the increasing numbers of pupils arriving at the senior high school level without this knowledge has created such an acute problem that many schools have felt the need of setting up remedial instruction in arithmetic.

In an effort to discover outstanding remedial programs that are in operation in senior high schools, the writer conducted during the spring of 1940 a nation-wide survey. Separately typed personal letters were sent to the 1090 principals of public high schools in the United States which are situated in towns of twenty thousand population or more. A number of questions were asked. Among them was the following: "What provision do you make for pupils who are deficient in the fundamentals of arithmetic, spelling, handwriting, English usage, and other tool subjects?" *The purpose of this article is to report information received from the schools relative to their remedial instruction in arithmetic.*

Three hundred seventy-nine schools in 38 states and the District of Columbia replied to one or more questions in the letter. Of these schools, 166 described their work in remedial arithmetic. An analysis of the data received indicates that the remedial work is generally handled in one or more of the following ways: (1) Remedial Arithmetic Classes, (2) Special Curricula for Pupils of Low Mental Ability, (3) General Mathematics Classes, (4) Special Arithmetic Classes for High School Seniors, (5) Courses in Commercial Arithmetic, Business Arithmetic, Shop Arithmetic, (6) Teachers in Regular Classes.

In the next few paragraphs the various methods and procedures used will be described in more detail, and specific illustrations from school practice will be presented.[1]

REMEDIAL ARITHMETIC CLASSES

Of the various plans in use for caring for pupils deficient in the fundamentals of arithmetic, those labeled "remedial arithmetic classes" are among the most common. In many cases, pupils are selected for these classes at the time they enter high school and in some cases before that time. Scores on arithematic tests often serve as the basis for admitting pupils to the classes. Grades made in arithmetic in the elementary school and the opinions of the elementary school teachers are also frequently considered.

Practice varies as to the length of time pupils remain in remedial arithmetic classes. Many schools require attendance for a full semester or year, while others expect the pupils to attend only a few weeks or until such time as they show sufficient improvement to enter the regular classes. Some schools admit pupils to the regular algebra classes and then after a test period of a few weeks remove those who are deficient in arithmetic and place them in special classes where they are taught the fundamentals they need. Most of the teachers of classes in remedial arithmetic gear the work to the interests, needs, and

[1] Thanks are due to Mr. Edward Mezner for valuable assistance in the preparation of this report.

maturation levels of the pupils in such classes.

In the following paragraphs are presented some direct quotations from the letters of schools which are carrying on classes in remedial arithmetic.

South High School, Omaha, Nebraska. "About ten years ago, South High School decided to offer a remedial program in mathematics. Too many students were found to be deficient in the fundamentals of arithmetic to handle algebra and geometry. These students were permitted to take algebra at first and after three weeks trial, those who could not learn algebra or geometry were transferred to Special Math. At present there are from 300 to 400 students who pass through this course each year. Special Math was developed to meet the actual needs of students. Therefore it starts as low as fifth grade arithmetic and goes through to advanced high school arithmetic."

George Washington High School, Indianapolis, Indiana. "We are doing some work that might be called remedial in arithmetic. We are inclined, however, to avoid the term remedial since this term implies that the schools preceding us have not done their task adequately. Our courses are designed to take students where we find them and we try to secure progress from this point. Our work in remedial arithmetic is still in the experimental stage. The first semester it was given, we gave mimeographed assignments reviewing the fundamental processes, mensuration, and percentage. We tried to carry out a program of individualized instruction but found that many of the difficulties encountered by the individuals applied to a large part of the class. We changed plans then and used a part of the class period to discuss with the class as a whole the underlying principles in solving problems. In the fundamentals we have insisted on a large amount of drill with weekly check-up on individual improvement. For two semesters we have been selecting drill work from Smith's "Workbook in Business Arithmetic." We have found much of the work too difficult but have selected work which we felt was needed to bring the student to the high school level in arithmetic ability. We have given the Woody-McCall Mixed Fundamentals Test as a means of determining their grade placement and as a means of finding the pupils' individual weaknesses."

Special Curricula for Pupils of Low Mental Ability

Brief descriptions of two schools caring for their remedial arithmetic through special curricula are given below.

South Philadelphia High School for Girls, Philadelphia, Pennsylvania. "When pupils fail badly with us in their first term, or any term thereafter, and their I.Q.'s are low, and when their teachers recommend it, we urge them to go into the Modified Course. Here we endeavor to maintain no particular standards but meet the pupils on their level even if it happens to be third or fourth grade. We give them work in fundamentals of arithmetic, spelling, handwriting, English, reading, and very simple writing. The girls in this course rarely return to the regular courses. They progress in the Modified Course and are graduated with the other girls, receiving certificates instead of diplomas."

Langley High School, Pittsburgh, Pennsylvania. "Three years ago we organized a class in our Junior-Senior High School which was comprised of all children who were at least two grades retarded, at least two years overage, and whose I.Qs. were below eighty. This group is being given three periods a day of so-called academic work including fundamentals of English, basic essentials in arithmetic, and practical social studies. During the rest of the school day these children take various shops and electives, such as art, music, and drawing. For arithmetic, I am using *Practical Problems in Arithmetic, Book Four,* published by the Webster Publishing Company, St. Louis. The problems in this arithmetic book, although practical, are so above the heads of some of the children that I shall have to find more simple material for next year. Of course, before the books were given to the children I cut out all references to the grade levels and have tried to make what we are doing with these children, 15–19 years of age, a dignified informal educational activity."

General Mathematics

Work in remedial arithmetic is handled by many schools through classes known as *general mathematics* or through classes of the same character known as *applied math-*

ematics, practical mathematics, or *everyday mathematics.* The content of such courses varies with the different schools, but practically all make provision for work in the fundamental processes of arithmetic. The procedures followed by four high schools in their general mathematics classes are sketched in the following paragraphs.

Emil G. Hirsch Senior High School, Chicago, Illinois. "This semester we have scheduled four classes in General Mathematics as part of our program of remedial instruction. The General Mathematics classes are offered in the 1B and 1A semesters. In all, 123 pupils are registered in General Mathematics. All entering 1B students whose total arithmetic score on the Chicago Survey Arithmetic Test is below 7.5 are placed in a General Mathematics 1 class. These pupils are given the New Stanford Arithmetic Test toward the end of the semester. Those with a score below 9.0 are given General Mathematics 2 the following semester. These General Mathematics classes are preparatory to the work in algebra for students with arithmetic deficiencies."

South Division High School, Milwaukee, Wisconsin. "The present course in General Mathematics derives its name from the fact that its main objective is the understanding and appreciation of cultural, social, and utilitarian applications of mathematics to every day living. While the student becomes acquainted with simpler algebraic concepts and geometrical terms and constructions, the major part of the course deals with mathematics (mostly arithmetic) applied to personal, home, and community situations and exemplified in measurements, budgets, scale drawings, per cents, insurance, graphs, money, taxes, buying, selling, and simple statistics. Every topic is vitally associated with arithmetic and necessitates computational assignments. The textbooks used are very modern and psychological in their presentations. Decidedly remedial work is offered every so often in challenging test work, followed by definite practice material on the particular type of skill found to require most remediation. The major part of the course is practically a repetition of eighth grade arithmetic, but the topics in the books are so well motivated, that the result is a real mathematical appreciation course, with remedial arithmetic pleasantly associated with mathematical notions which the pupils can really understand and honestly enjoy. The books found very satisfactory for this course are *Mathematics in Life* by Schorling and Clark and *Living Mathematics* by Ruch, Knight, and Hawkins."

Dallas Technical High School, Dallas, Texas. "We have a two year general mathematics program. All pupils who are weak in the freshman year in mathematics are required to take general mathematics 1 and 2, and then they may elect the regular sequence of algebra, geometry, trigonometry, or they may continue with the general mathematics for the second year. This second year of general mathematics is applied and technical mathematics for the non-college boy."

Beaumont High School, St. Louis, Missouri. "Many students are promoted to high school who are not prepared to take formal algebra. Until recently, algebra was required of all our entering pupils. At present, a year of mathematics is required and while theoretically our pupils may choose between a year each of algebra, business arithmetic, or practical mathematics, we do not let it work out that way in practice. In practice, we give all boys and girls entering high school a test called "The St. Louis Test of the Fundamentals." A perfect score on this test is 44, and we find that if a student does not make 31 or more he has very little chance of making satisfactory progress in formal algebra. Hence, we divide the algebra classes very soon after formation and we find by experience that from $\frac{2}{3}$ to $\frac{3}{4}$ of those who choose algebra are prepared to take it. We actually place the other third or fourth in Practical Mathematics. We do not encourage those who take Practical Mathematics to take algebra after the year of Practical Mathematics. We apply the same criterion to those who sign for Business Arithmetic to determine whether they may take Business Arithmetic or must take Practical Mathematics. In addition to the score of 31 points out of 44 or more, we use the ranking of the student in his elementary class at graduation and we use his comparative rating in arithmetic in elementary school."

Central High School, Omaha, Nebraska. "Two years ago we began offering a course in General Mathematics. Students may elect this course at the time of their entrance, or they may be advised to take it after they have been given some tests,

or perhaps, even after they have tried algebra for a while. The first semester of General Mathematics, as we teach it, deals very largely with the fundamentals of arithmetic. A second semester of General Mathematics may be elected by the student, or he may go into algebra, depending upon his plans."

Arithmetic Classes for High School Seniors

Realizing that a knowledge of the fundamentals of arithmetic is essential for success in adult life, a number of schools have set up arithmetic classes for those high school seniors who are about to graduate and who have as yet failed to master these fundamentals. Such courses are usually called *senior arithmetic* or *arithmetic review*. Excerpts from some of the letters will illustrate methods and procedures employed in these classes.

Benjamin Franklin High School, Los Angeles, California. "The plan for arithmetic review is just beginning to operate. All A-11 pupils take the Los Angeles Fundamentals of Arithmetic Test. The tests are then checked by the mathematics teachers. The results are plotted together with the I.Q. of each pupil. Passing or failing in this test is determined by relating the test score to the ability level of the pupil as shown by the I.Q. For example, pupils of 100 I.Q. or over must show a grade placement level of 7.5 or better. Pupils of seventy or less I.Q. are not considered to have failed the test whatever their score. All pupils who have failed this test are then advised to substitute ten weeks of a course in Arithmetic Review for their required ten weeks in Senior B of Consumer Economics or Social Arts, or if they desire a full semester of arithmetic review they may enroll in General Mathematics B-10. Even those who pass the test may take the ten-week review in arithmetic if they wish to do so. The taking of remedial work in arithmetic is voluntary, at least thus far. But if any pupil is advised to take remedial work because of his failure in the test, and does not do so, a notation of 'deficiency in arithmetic' is made on his permanent record card, his last report card, and any transcript of record that is issued."

Arsenal Technical Schools, Indianapolis, Indiana. "A small pamphlet called 'The Fundamentals of Arithmetic' produced locally in mimeographed form is being used in all classes below Algebra III for remedial purposes. This pamphlet consists of ten lessons, which emphasize the fundamentals of arithmetic. Members of the senior class who are about to graduate are given voluntary instruction on the material in the above named pamphlet and are tested on the material. The grades made on these tests are incorporated into the permanent records of the school."

Kern County Union High School, Bakersfield, California. "Seniors found deficient in arithmetic through use of a standardized test are required to take a six weeks' review of these processes without credit and must pass with a minimum standard to eligible for graduation."

Western High School, Baltimore, Maryland. "We have remedial arithmetic classes for twelfth grade students who fail to make 80% in a test on arithmetic fundamentals. As soon as pupils are able to pass such a test, they are excused from further attendance in the class."

Commercial Arithmetic, Business Arithmetic, Shop Arithmetic

Although not labeled remedial, much remedial work in arithmetic is frequently carried on through courses designated as commercial arithmetic, business arithmetic, and shop arithmetic. For example, in Castlemont High School, Oakland, California, "students who are poor in arithmetic may elect business arithmetic." At the Santa Barbara (California) High School, students sign up for "Commercial Arithmetic" if their arithmetic standing is below the 30 percentile on the Progressive Achievement Test in Arithmetic.

Teachers in Regular Classes

Some schools try to handle the problem of extreme retardation in arithmetic through the regular mathematics classes. The following quotation illustrates this practice: "We have no special classes for remedial work in arithmetic. The various handicaps are diagnosed by the teachers of mathematics and remedial work is applied in the regular classes."

In addition to such work, many teachers give assistance to pupils who are deficient in arithmetic during open periods or before and after school hours.

Schools using such techniques, as a rule, stated their dissatisfaction with the results and expressed a hope that more specific and better organized work might soon be initiated.

Summary and Conclusions

This paper briefly summarizes some of the methods and procedures used by 166 senior high schools for dealing with retardation in arithmetic. The data were gathered in 1940 as a part of a nation-wide study of remedial teaching in secondary schools. It is evident from the reports received that increased attention is being given to organizing programs which will care for those children at the high school level who have failed to master the fundamental processes of arithmetic. The modern secondary school is obviously striving more and more to meet each child on his own ground—to take him where he is, and from that point to assist him in advancing toward greater competence in the use of the essential tools of education.

Mathematical Theory of the Mercator Map

By NELSON B. CONKWRIGHT
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BECAUSE of the attention which the Mercator projection has recently received in the press, the manner in which this map is used by the navigator is doubtless familiar to a large section of the public. The projection owes its importance to the fact that the rhumb line direction between two points on the earth can be found by drawing a straight line between the corresponding points on the map, and measuring the angle at which the line intersects the meridians of the map.

But a theoretical discussion of this projection, suitable for students and teachers of mathematics, does not seem to be available. If the standards of rigor demanded are not too exacting, the theory can easily be developed. The details are as follows.

Assume that the earth is a perfeet sphere, and denote its radius by r. In Fig. 1 the point N is the north pole, O is the center of the earth, EQ is an arc of the

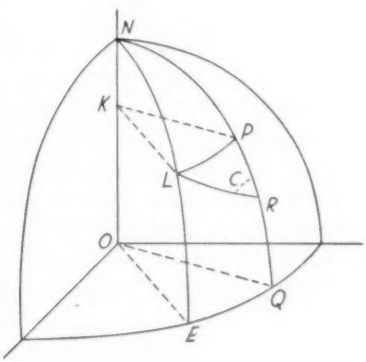


FIG. 1

equator, and PL is an arc of a parallel. Both NE and NQ are meridians. The curve RL represents a rhumb line cutting the meridians at the constant angle C. If θ and ϕ denote the latitude and longitude respectively of points on the rhumb line, then this curve determines ϕ as a function of θ.

Let the latitude and longitude of the fixed point R be θ_1 and ϕ_1 respectively. Denote the angle at O subtended by the arc RP by $\Delta\theta$, and the angle at K subtended by the arc PL by $\Delta\phi$. These angles are understood to be measured in radians. Then the radius KP of the circular arc PL is $r\cos(\theta_1+\Delta\theta)$, and the length of arc PL is $r\cos(\theta_1+\Delta\theta)\Delta\phi$. The length of arc RP is $r\Delta\theta$. Thus it is evident that

$$\tan C = \lim_{\Delta\theta\to 0} \frac{r\cos(\theta_1+\Delta\theta)\Delta\phi}{r\Delta\theta}$$

and therefore

$$\tan C = \cos\theta_1 \frac{d\phi}{d\theta}.$$

Since R was understood to be any point on the rhumb line, we may drop the subscript on θ, and affirm that at any point (θ, ϕ) on the curve we have

$$\tan C = \cos\theta \frac{d\phi}{d\theta}. \tag{1}$$

The reader is cautioned against a misinterpretation of the statement that $\Delta\theta\to 0$. The intention is that the point L shall move along the curve toward the point R. The rhumb line itself is a *fixed* curve.

Now in the plane map (Fig. 2) the line $E'Q'$ represents the equator, and the meridians are the lines perpendicular to

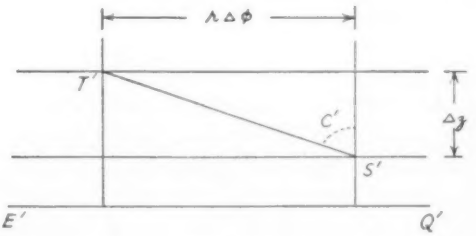


FIG. 2

$E'Q'$. The meridians in the map are so spaced that the distances between them

are equal to the corresponding distances on circle EQ (reduced to scale). The lines perpendicular to the meridians represent parallels. The manner in which they are spaced will now be explained.

Let S and T be any two points on the rhumb line, and draw a straight line between the corresponding points S' and T' of the plane map. The parallels in the plane must be spaced in such a manner that the line $S'T'$ will intersect the meridians at the angle C. In other words, it is required that the angle C' in Fig. 2 be equal to angle C on the sphere.

To effect the desired spacing of the parallels, we take the linear distance representing one minute of longitude along $E'Q'$ as the unit of length. This unit of length will be called a *meridional part*. Let z denote the distance on the map from the equator to the parallel of latitude θ. The line $S'T'$ determines z as a function of ϕ. Let ϕ be measured in radians. Then

$$\cot C' = \lim_{\Delta\phi \to 0} \frac{\Delta z}{r\Delta\phi} = \frac{1}{r}\frac{dz}{d\phi},$$

where of course r is measured in meridional parts. It is now clear that C' will be equal to C if

$$\sec\theta \frac{d\theta}{d\phi} = \frac{1}{r}\frac{dz}{d\phi}.$$

It follows that

$$dz = r \sec\theta \, d\theta$$

and therefore

$$z = r \log_e \tan(45° + \tfrac{1}{2}\theta). \qquad (2)$$

The constant of integration is zero since $z = 0$ when $\theta = 0$.

Formula (2) is the relation which determines the spacing of the parallels on the Mercator map. In modern tables for the values of z, minor corrections have been effected to allow for the fact that the earth is not a perfect sphere. The corrections are relatively small.

It may be of interest to consider how the distance scale is determined in the actual construction of a map. Let it be required to construct a map covering all 360° of longitude, and having an east-west extent of 20 inches. Then it will be convenient to think of the map as a representation of a terrestrial globe with a circumference of 20 inches.

If the meridians of the map are drawn at intervals of 10°, the distances between consecutive meridians will be 20/36 in. The radius of the globe is $10/\pi$ inches. The length in inches of a meridional part can easily be determined. Using the meridional part as the unit of length, it will be found that

$$r = \frac{360 \times 60}{2\pi}.$$

The distance from the equator to the parallel of latitude 30° N (for example) will be

$$\frac{360 \times 60}{2\pi} \log_e \tan(45° + 15°)$$

meridional parts.

In practice we first decide upon the latitudes to be represented, and the size of the sheet of drawing paper to be used. The actual length of a meridional part is then governed by the necessity of getting the desired latitudes upon the paper.

Recent manuals dealing with the Mercator map have naturally emphasized the fact that distances are grossly exaggerated on this projection. It is not so generally known that there is a simple formula for the distance along a rhumb line. If h denotes the difference of latitude (in minutes of arc) between two points on the earth, the rhumb line distance between them is $h/\cos C$.

The formula for the distance can be derived as follows: In Fig. 1 let the length of the arc RL of the rhumb line (in nautical miles) be denoted by Δs. If $\Delta\theta$ denotes the distance RP in nautical miles, it is clear that

$$\cos C = \lim_{\Delta s \to 0} \frac{\Delta\theta}{\Delta s} = \frac{d\theta}{ds}$$

and

$$ds = \frac{d\theta}{\cos C}.$$

Then the distance between two points with latitudes (in minutes) of θ_1 and θ_2 is given by the formula

$$s = \frac{\int_{\theta_1}^{\theta_2} d\theta}{\cos C} = \frac{\theta_1 - \theta_2}{\cos C}.$$

We have yet to show that, if the line $S'T'$ on the Mercator map passes through the point whose latitude and longitude as read from the map are θ_3 and ϕ_3 respectively, then the rhumb line actually passes through the corresponding point (θ_3, ϕ_3) on the sphere. To dispose of this detail, integrate (1), and obtain the relation

$$\phi = \tan C \log_e \tan (45° + \tfrac{1}{2}\phi) + M, \tag{3}$$

M being a constant of integration. Relation (3) may be regarded as the equation of the rhumb line in terms of the coordinates θ and ϕ. The quantities C and M can be determined by requiring that the rhumb line pass through two specific points on the sphere.

It follows from (3) that

$$w = z \tan C + M \tag{4}$$

where $w = r\phi$. Relation (4) may be regarded as the equation of the line $S'T'$ with reference to a suitably chosen system of rectangular coordinates in the plane. By means of (3) and (4) it is easy to establish the desired correspondence between points on $S'T'$ and points on ST.

Questionnaire Sent by Sub-committee of Mathematics Section to Eastern Colleges*

1. What per cent of the Scholastic Aptitude Test do you weigh the Mathematics Section?
2. Do you require an achievement test in Mathematics of the Alpha, Beta or Gamma type in addition to the Scholastic Aptitude test?
3. Do you provide alternate courses for entering Freshman on the basis of their achievement in the Scholastic Aptitude Test? . . . on the basis of the results on achievement tests?
4. Do you permit entering students, with four years of adequate preparatory work, to enter Sophomore Mathematics?
5. If your answer to question four is *no*, what Freshman course are they permitted to enter?
6. Has the achievement on the Mathematics section of the Scholastic Aptitude Test compared favorably, in the past, with the students' marks in the Freshman year at college?
7. Will students be rejected because the mark is low on the Mathematics Section of the Scholastic Aptitude Test if the school record is good?
8. Have you any definite suggestions as to the curriculum in secondary school Mathematics which would be valuable for college courses?

* The above questionnaire was the one sent to colleges which used the C.E.B. examinations, or are of particular interest to private schools by the Private School Teachers Association of New York. The Report on the replies to this questionnaire was published on pages 317–320 of the November (1943) issue of THE MATHEMATICS TEACHER. The report can be better understood by referring to the above questionnaire—Editor.

The Teacher of Mathematics and the War Savings Program*

Including Problems for Elementary and High School Classes in Mathematics and Business Arithmetic

How Can the Teacher of Mathematics Help?

Teaching is a challenge at any time. In wartime the challenge increases with the problems which grow out of the current crises. Today as teachers endeavor to develop the practical skills needed by their pupils, they face the growing challenge of encouraging new and broader understandings and attitudes. For the student of mathematics is a citizen in a Democracy where he will take his place as a builder of the peace if he is well-informed and aware of his responsibilities. How can the teacher of mathematics contribute to such an objective? What *more* can he do in the hectic demands of crowded classes and increasing assignments?

The teacher of mathematics may contribute to this objective through the classroom study of current financial and economic problems as they relate to the individual citizen. The problems of saving, borrowing, and budgeting are all part and parcel of the typical study of mathematics in the elementary and secondary school. To approach these problems in the light of current facts and figures may contribute to an increased feeling of responsibility on the part of students themselves.

In many schools pupils have worked out arithmetic problems in terms of their own War Savings activities. Adding their Stamp purchases in the elementary grades has been more exciting than merely adding a prepared column of dead figures. Estimating the purchasing power of their combined War Savings investment in terms of military equipment has made the question of financing the war more vital and personal. At the same time these projects have provided sound learning situations.

This bulletin, "The Teacher of Mathematics and the War Savings Program," has been prepared to assist teachers in developing the classroom study of wartime financial problems. It is hoped that the introduction of such problems will increase pupil understanding of the current savings program. From increased knowledge will come greater cooperation on the part of students and their parents in the voluntary savings program of the country.

Part One—What Understandings and Attitudes May be Attained?

About the Wartime Needs of the Government

I. Material Needs.—For our armed forces we need vast numbers of ships, tanks, planes, and quantities of ammunition, clothing, food, and many of the articles that civilians use in their daily life.

II. Financial Needs
   - A. The total expense for conducting the war is enormous, expressible only in huge numbers formerly used only in connection with the study of astronomy.
   - B. The Government gets the money to pay its expenses either by collecting taxes from the people or by borrowing money from individuals, corporations or banks.

III. Money From Taxation
   - A. Certain *direct taxes* are collected including income taxes, social security taxes, inheritance taxes, gift taxes, and taxes on luxuries.

* *A Schools at War Bulletin.* Prepared by Walter W. Hart, Veryl Schult, and Violet Coldren. With Members of the War Finance Division.

Published by the Education Section of the War Finance Division, U. S. Treasury Department, Washington 25, D. C.

B. Certain *indirect or hidden taxes* are a part of the price of every article —the part that the manufacturer or retailer of the article must add in order that he can pay the direct taxes imposed on his business.

C. The Government must try to make the tax collected from each person fair and reasonable, by adjusting it to the benefits received by the person and to his ability to pay.

D. Taxes have increased greatly for all the people and must increase still more.

E. Taxes are the price we pay for the services rendered by the Government. It is patriotic to pay them willingly.

IV. Money Borrowed by the Government

A. The Government cannot collect through taxation all the money it needs for wartime expenses. It must borrow a part of what it needs.

B. The Government gives to the lender a printed promise to repay the amount borrowed from him. A bond is such a promise. At present, most of the bonds sold to individuals are called War Savings Bonds. To those who lend less than $18.75 at one time, the Government gives War Savings Stamps.

C. The Government pays "rent" or interest for the use of money borrowed through the sale of Bonds. It pays no interest on Stamps which should therefore be converted into Bonds as soon as possible.

D. A United States War Bond is the safest investment in the world.

E. It is patriotic to lend money to the Government.

V. War Bonds and Stamps

Detailed information about them appears in a booklet entitled *Questions and Answers about United States War Savings Stamps and Bonds*, issued by the Treasury Department as form #WSS 490. A copy of this booklet should be in each classroom, and the principal facts in it should be taught to the pupils, according to their maturity. Emphasize especially:

A. Buying Bonds is lending money to the Government, not giving it.

B. It will help the Government most and will be of advantage to the lender if the Bonds are kept for the ten years it takes for them to mature.

C. In case of emergency, Bond owners may redeem any War Bonds held 60 days or longer. Full purchase price will be paid, and for Bonds held over a year interest will be paid as well.

D. All citizens must save as much as possible out of current income for the purchase of Stamps and Bonds. This will require careful planning, constant thrift, and a willingness to do without peacetime luxuries.

About Inflation

I. The vast purchases by the Government for the armed forces

A. Have reduced the quantity of food and supplies available for civilians.

B. Have increased the number of people needed to raise food and manufacture articles for the armed forces.

C. Have increased greatly the incomes of many people and the total income of the public as a whole.

II. Prices have increased

A. Because the quantity of available goods has decreased.

B. Because the wages paid to produce the goods have increased.

C. Because the amount of money with which people as a whole can buy has greatly increased.

III. Inflation

A. Results from the facts stated above.

B. Destroys the benefits of increased wages because of increased prices.
C. Is ruinous for those whose income has not increased.

IV. 7-Point Anti-Inflation Program (outlined in the President's message to Congress on April 27, 1942)
A. We must, through heavier taxes, keep personal and corporate profits at a low reasonable rate.
B. We must fix ceilings on prices and rents.
C. We must stabilize wages.
D. We must stabilize farm prices.
E. We must put more billions into War Bonds.
F. We must ration all essential commodities which are scarce.
G. We must discourage installment buying, and encourage paying off debts and mortgages.

V. War Bonds vs. Inflation
A. Money not needed to buy necessities must be saved for War Bonds.
B. Money saved now and loaned to the Government can be spent after the war is over
1. When goods will be more plentiful, prices lower, and the quality probably higher.
2. When some luxuries will be available which cannot be bought now, such as automobiles, radios, and refrigerators.

About Planned Savings

I. People are more likely to save money if they definitely plan to do so.

II. Such a plan is called a budget
A. Since the cost of necessities is up, the allowance for necessities must be larger than before the war.
B. Since many luxuries cannot be bought now, the money spent for such luxuries before the war can be saved now.
C. The new taxes collected by the Government reduce the income people have to spend now.
D. Employees may authorize a certain allotment from each paycheck to be saved for the purchase of Bonds. This is known as the Payroll Savings Plan whereby the employer delivers the Bond to the employee out of designated savings from his paycheck.

III. Money saved now and invested in Bonds will be returned by the Government. If held for ten years, each Series E Bond will earn one dollar for every three invested.[1]

Part Two—War Savings Problems for Grades III Through VI

Introduction

The sale of War Savings Stamps in classrooms is begun much earlier than at the third grade level. Even kindergarten children can begin to understand that the Government must have a "lot" of money to pay for war materials. First and second grade children can understand that their purchase of War Savings Stamps helps the Government to pay for the expenses of the war.

Since many schools do not have a planned arithmetic program before the third grade, the following examples and problems are suggested for use in Grades III through VI. Many more such problems will be suggested by them and by the discussions carried on in the various classrooms in regard to the purchase of War Savings Stamps and Bonds. Additional problems can be made easily from the "Facts and Figures" found in the Appendix, pages 373–374.

Examples are grouped according to the mathematical process needed in their solution. In each group, examples are arranged in the order of approximate difficulty or grade level.

Thus, in the group listed under "Numeration," examples 1 and 2 may be used in the third grade, examples 3 through 8 may be used in Grades IV and V, and ex-

[1] "Facts and Figures for Making Problems" are given in the Appendix. See pages 373–375.

amples 9 and 10 may be suitable for Grade VI.

Numeration

Read the following sentences:

1. A seaman's white hat costs $.60
2. A set of arm insignia costs $.10
3. Four cotton handkerchiefs cost $.20
4. A pair of gloves costs $.99
5. An arm splint costs $1.65
6. A rifle costs $85.00
7. A submachine gun costs $55.00
8. A jeep costs $1165.00
9. A small pursuit plane costs in all $75,000.00
10. An ambulance plane costs in all $110,000.00

Meaning of Numbers

The small child has had little experience in handling money, recognizing coins, or making change. Similarly, he is unacquainted with the cost of supplies and equipment needed for the armed services. The following examples are intended to broaden his conception of the meaning of numbers and the value of money and goods:

1. How many pennies must you save to buy a 10¢ Savings Stamp?
2. How many nickels must you save to buy a 25¢ Savings Stamp?
3. How many 10¢ Stamps must you buy to have enough money with which to buy a Bond that costs $37.50?
4. It costs 30¢ to feed a carrier pigeon for one month. How many pupils must buy a 10¢ Stamp to feed a pigeon for a month?
5. A set of signal flags for a destroyer costs $125. How many pupils must buy a 10¢ Stamp to lend Uncle Sam enough money to pay for such a set of flags?
6. $1500 pays for one motor trailer. How many 25¢ War Stamps must be bought to pay for a trailer?
7. How many 25¢ Stamps must be bought by each pupil in your class in order to pay for a motor trailer?
8. A jeep costs $1165.00. If your school purchases a jeep during the coming month, how many 25¢ Stamps must the pupils of the school buy? How many stamps is this per pupil?
9. The cost of a 37 mm. anti-tank gun is $3400. How many Bonds costing $18.74 must be sold by the Government to pay for such a gun?
10. A light tank costs $45,000. How many 25¢ Stamps must be bought to provide Uncle Sam with enough money to pay for such a tank? If every pupil in our school buys a 25¢ Stamp every week, how long will it take us to "buy" a light tank?

Addition

1. Every sailor is allotted certain items of clothing. What is the total cost of those listed below?

1 comb	$ .05
1 pair of wool socks	.35
1 black belt	.20
1 pair of gloves	.85
1 pair of shoes	4.25
1 pair of woolen trousers	7.00
1 woolen shirt	5.25
	$

2. For a soldier the cost of clothing differs slightly. What is the total cost of the following items?

1 woolen coat	$11.53
2 woolen trousers	12.38
1 overcoat	15.50
2 pairs of shoes	8.62
3 khaki shirts	6.96
1 garrison cap	1.03
1 cotton cap	.63
	$

3. It costs $200.00 to clothe a Marine; $135.00 to clothe a sailor; $127.00, a WAC; $189.00, a WAVE; and $189.00 a SPAR. What is the total cost of these outfits?
4. Army invasion equipment is expensive. What is the sum of the cost of the following items?

1 amphibian jeep or "quack".	$ 2,090.00
1 invasion barge	12,500.00
1 60 mm. mortar	315.00
1 37 mm. anti-tank gun	3,400.00
1 40 mm. anti-aircraft gun	25,000.00
	$

5. What is the total cost of the following items used in rescue work by the Navy?

1 stretcher	$ 17.20
1 ambulance plane	110,000.00
1 aircraft rescue boat	20,000.00
1 self-inflating life belt	4.75
1 life float	150.00
	$

Subtraction

1. You have saved 7¢. How many more pennies do you need before you can buy a 10¢ Savings Stamp?
2. You have saved 18¢. How much more must you have before you can buy a 25¢ Stamp?
3. You have 36¢. How much will you have left after you buy a 25¢ Stamp.
4. Someone gave you $1.00 for your birthday. Of that you have spent 35¢ for a pair of socks.
   a. How much is left out of your $1.00?
   b. How much more must you have before you can buy one 50¢ Stamp and two 25¢ Stamps?
5. A $25 Bond costs only $18.75. This Bond is worth $25 ten years from the month in which it is bought. What is the difference between the cost of the Bond and its value at the end of 10 years?
6. The $50 Bond costs $37.50.
   a. When does this Bond have its full value?
   b. What is the difference between the cost of the Bond and its full value?
7. Mr. Wilson formerly spent about $24 a month for gasoline for his car. Now that gasoline is rationed, he spends only $4 a month for gasoline.
   a. How much that he formerly spent for gasoline can he save now?
   b. What kind of War Bond can he buy each month with the money he saves?
   c. After buying the Bond, how much cash will he have left?
8. A jeep costs $1165. An amphibian or "swimming" jeep costs $2090. What is the difference in the cost of these two cars?
9. A dive bomber can be bought for $110,000 and a small fighter plane for $75,000. How much more than the cost of the fighter plane is the cost of the dive bomber?
10. Our school is planning to buy a jeep which costs $1165. We have already sold $622 worth of Stamps. How many more 25¢ Stamps must we sell before we can pay for our jeep?

Multiplication

1. A sailor's comb costs 5¢. What is the cost of such combs for nine sailors?
2. The cost of a cartridge belt is 83¢. How much does it cost Uncle Sam for belts for 125 men?
3. 27¢ pays for the first-aid pouch that each man must have. What does it cost for such pouches for 75 men?
4. A man decided to give bonds to his family for Christmas presents instead of spending the money for the usual kinds of gifts.
   a. He bought an $18.75 Bond for each of his 3 children. How much did that cost him?
   b. If he also bought a $37.50 Bond for his wife, what was the total cost of all of the Bonds that he bought?
5. One parachute costs $65 if made out of Government-owned silk. How much does it cost to supply parachutes for 200 paratroopers?
6. Twenty-five boys who graduated from our school have just entered the Navy. Their clothing costs $135 apiece. The pupils of the school have

decided to lend Uncle Sam enough money to pay for the needed clothing for these boys. How many 25¢ Stamps must they sell?

7. Four young men from our neighborhood volunteered for service with the Marines. How many $18.75 Bonds must the school sell to provide Uncle Sam with enough money to pay for the clothing and equipment for these Marines, since it costs $200 for each of them?
8. 60¢ a day feeds an enlisted man of the Army while he is in the United States. Find the number of men from your town who are in the army and estimate the cost of feeding them one day.

Division

1. One day's food for a war dog costs about 30¢. How many 10¢ Stamps must you buy to lend Uncle Sam enough to feed a dog:
   a. For one day? b. For one month?
2. How many dogs could Uncle Sam feed with the money which your class saves regularly?
3. How many 25¢ Stamps will provide enough money to pay for an $18.75 Bond?
4. $65 pays for a parachute. One member of the class has bought an $18.75 Bond with money he has earned. How many 25¢ Stamps must be bought by the class to provide the rest of the money needed to pay for a parachute?

Fractions

Edward Jones now earns $60 a week, whereas before the war he earned only $38 a week.

1. How much more are his weekly wages now than before the war?
2. Out of his $60 he must pay an income tax to help pay the cost of the war. His employer may figure his tax as follows:
   a. From the wages earned, he subtracts $12 which is his tax exemption. He must pay a tax on all except this $12. On what remaining sum must he pay a tax?
   b. To pay this income tax Edward's employer deducts one-fifth of this remaining income. How much is taken out of Edward's wages each week for income tax?
   c. How much is left for Edward each week after the income tax has been subtracted from his total salary?
   d. Edward asks his employer to take out one-fourth of the remainder left after doing Example (c) to be spent for buying War Bonds. How much does Edward agree to save each week for the purchase of Bonds and Stamps?
   e. Now how much does Edward have left out of his week's wages?

Planned Savings

The average person should plan his savings in advance. Then he is sure to save in proportion to his income and obligations.

1. Ted Armstrong earns a dollar every Saturday at the neighborhood grocery. With this he pays 5¢ for milk each day at School. The rest he puts in War Savings Stamps. How many Stamps can he put in his 25¢ Stamp Album each week?
2. Ted's father earns $40 a week. He decides he will save one-eighth of his wages for War Bonds.
   a. How many 50¢ Stamps will he buy each week?
   b. How much money will he save during the year?
   c. How many Bonds costing $18.75 will he buy during the year?
3. Dr. Smith always saves $10 each month to pay for a summer vacation trip for the family. This year the Smiths have decided to spend their vacation at home and put the money into War Bonds.
   a. How many $18.75 Bonds will they be able to buy in a year?

b. How much money will be returned to them ten years from now for their postponed vacation trip?

4. Keep a record of the money which you earn, receive as a gift, or which is spent for your food, clothing, school equipment and recreation.
   a. How much of this did you save and invest in War Stamps?
   b. How much more could you have saved by doing without certain luxuries, such as extra candy or movie tickets?
   c. What additional savings could you have made by more careful handling of your school supplies, your shoes, clothes, and personal belongings?

General Problems

1. Suppose that your school decides to sell enough Stamps and Bonds to pay for a flying jeep or Grasshopper plane ($3000).
   a. How many 10¢ Stamps must be sold to collect the money needed?
   b. How many 25¢ Stamps would provide the needed money?
   c. How many Bonds costing $18.75?
2. In the two months following the bombing of Pearl Harbor the pupils of the schools in Hawaii bought $824,000 worth of Stamps and Bonds.
   a. Since the clothes and the equipment for a Marine cost $200, how many Marines can be equipped and clothed with this money?
   b. How many $85 rifles could that amount of money buy?
3. Find the number of Bonds costing $18.75 that your school must sell to help Uncle Sam furnish the clothing and equipment for 115 soldiers who graduated from your school. Remember the cost of the clothing and equipment for one soldier is about $165.
4. Your school may name a plane if it sells enough Stamps and Bonds to pay for the plane. Suppose you decide to "buy" a small pursuit plane that costs $75,000. How many Bonds costing $18.75 must you sell to raise that much money?
5. In one class there are twelve pupils, each of whom has a big brother in the Army.
   a. How many 25¢ Stamps must that class buy to pay for a gas mask and a first-aid kit for each of the big brothers if a gas mask costs $9.25 and a first-aid kit costs $4.89?
   b. How many $18.75 Bonds must be bought to pay for an overcoat for each if an overcoat costs $15.50?
6. After 10 years Uncle Sam gives as "rent" or interest the difference between the cost of a Bond ($18.75) and the full value of the Bond which is $25. What is the interest for the 10 years?
7. Find the "rent" for 10 years paid by Uncle Sam for the $37.50 which is the cost of a $50 Bond. A $100 Bond which costs only $75.
8. Assume that $1200 will buy a fine car eleven years from now.
   a. How many $25 Bonds must your family buy during the coming year in order to have enough money to pay for such a car eleven years from now?
   b. What will the Bonds cost your family now?[2]

Part Three—War Savings Problems for Grades VII and VIII

Introduction

Before proceeding with the suggested classroom activities for grades VII and VIII, teachers are urged to examine carefully the material for Grades III through VI in Part Two. Similar examples can be prepared for use in grades VII and VIII

[2] For additional "Facts and Figures for Making Problems" See Appendix on pages 373–375.

during that part of the year normally devoted to a review. Such examples should use the larger numbers that appear in the Appendix, "Facts and Figures for Making Problems" (pages 373–375) and may be more complicated.

Frequently seventh and eighth grade pupils are wage earners after school and on Saturday. Many of them are not only handling their own earnings but are assuming an increasing share in family shopping, budgeting, banking and economy. Their careful purchasing and thoughtful saving may be the most steadying influence in a family of new wage earners untrained in the rudiments of budgeting and banking. Thus the classroom lessons in thrift and planned economy assume a new significance for the student as an individual and as a member of a family unit.

Numeration in Grades VII and VIII may well be limited to the millions.

(A) The meaning of one million should be taught by some dramatic example. For example: A dollar bill is $6\frac{1}{8}$ inches long. If a million of them were laid end to end, they would cover 6,125,000 inches or approximately 97 miles! Or: If a dollar were spent every minute of the day, it would take about two years to spend a million dollars!

(B) Another way of writing $5,800,000 is $5.8 million.

Examples and Problems in Reading and Writing Numbers

1. Federal tax revenues may be increased from $35,000,000,000 to $51,000,000,000 during the fiscal year 1944 (July 1, 1943 through June 30, 1944).
2. In the fiscal year 1944, the war will cost at least $100,000,000,000, the equivalent of about $2,500 for each family in the country.
3. Modern warfare is expensive. Some costs follow:
   A life raft for 25 persons costs $2650.
   A 37-millimeter anti-aircraft gun costs $12,325, and a 155-millimeter Howitzer costs $32,700.
   Two-engined bombers cost $175,000 while a Flying Fortress costs $450,000.
   Cruisers cost between $35,000,000 and $77,000,000.

Percentage

Percentage problems take on a new meaning when they are based on the vital cost of military equipment, school War Savings reports, and personal savings records.

I. Jefferson Junior High School has had a continuing War Stamp and Bond campaign. Realizing that winning the war depends on every citizen, and that each has a responsibility in the War Savings campaign, Jefferson pupils bought $3574 worth during February.
   1. 1150 pupils participated. What was the average amount invested by each pupil for the month? The average per week?
   2. If one jeep costs $1165, how many jeeps would the total amount buy?
   3. a. If these students bought only $2950 worth during January, how much more did they buy in February than in January?
      b. The increase is what per cent of the January purchases?
      c. The increase is what per cent of the February purchases?
   4. The cost of a 37-millimeter anti-aircraft gun is $12,325. The Jefferson Junior High School February investment represents what per cent of the cost of one such gun?
   5. The Jefferson pupils invested $4155 in War Bonds and Stamps during March.
      a. What was the total investment for the 3 months of January, February and March?
      b. The March investment is what per cent of the total for these three months?

c. A Navy PT boat costs about $145,000. The Jefferson investment for these three months is what per cent of the cost of such a boat?

II. From your school or class War Savings Committee find the total Stamp and Bond sales for your school and the total enrollment.
1. What was the average amount invested by each pupil for the past month? The average per week?
2. How does the weekly average compare with the saving of the Jefferson Junior High School in Problem I above?
3. What pieces of military equipment could be financed by your War Savings purchases?
4. What per cent of your total enrollment actually bought Stamps or Bonds each month? Each week? How does this compare with other classes in your school or other schools in your community?

III. Mr. Jackson earns a salary of $3200. He invests 15% in War Bonds. Mr. Thomas, who has a smaller family and earns $2700, invests 18% in War Bonds. Who invests the larger sum in War Bonds?

IV. In 10 years, a Bond bought for $37.50 will have a value of $50.
1. How much money will the investment earn in 10 years?
2. What is the average amount earned each year?
3. This average amount is what per cent of the amount invested?

V. An Army jeep costs $1165 while an amphibian jeep or "quack" costs $2090.
1. The cost of the "quack" is how much more than the cost of the jeep?
2. The cost of the jeep is what per cent of the cost of the "quack"?

Budgeting

Thrift means the wise use of one's money. This can be accomplished only through careful planning or budgeting. In these days when incomes are high and consumer goods are scarce, a larger proportion of your income can be saved. Every available dollar is needed by the Government for the purchase of military equipment and supplies. Your dollars can go to war to support your fathers and brothers on the battle front.

There was a time when people saved what was left over after all expenses were paid. Today, patriotic citizens set aside from 15% to 25% of their earnings for War Bonds and then plan the rest of their budget accordingly. Check up on your own income and necessary expenditures. Then figure it out yourself.

The first budget plan below is a good one:

For:	War Bonds	Shelter	Food	Clothing	Insurance and Operating Expenses	Health Education and Advancement	Taxes
Budget:	20%	20%	20%	10%	10%	10%	10%

Note: Taxes will amount to 10% of small incomes; they will be higher in larger incomes.

For:	War Bonds	Shelter	Food	Clothing	Insurance and Operating Expenses	Health, Education and Advancement	Taxes
Budget:	$3	$15	$16	$18	$9	$2	$7

I. How much will be spent each month for each purpose if the monthly income is:
   1. $100? 2. $125? 3. $200? 4. $175?

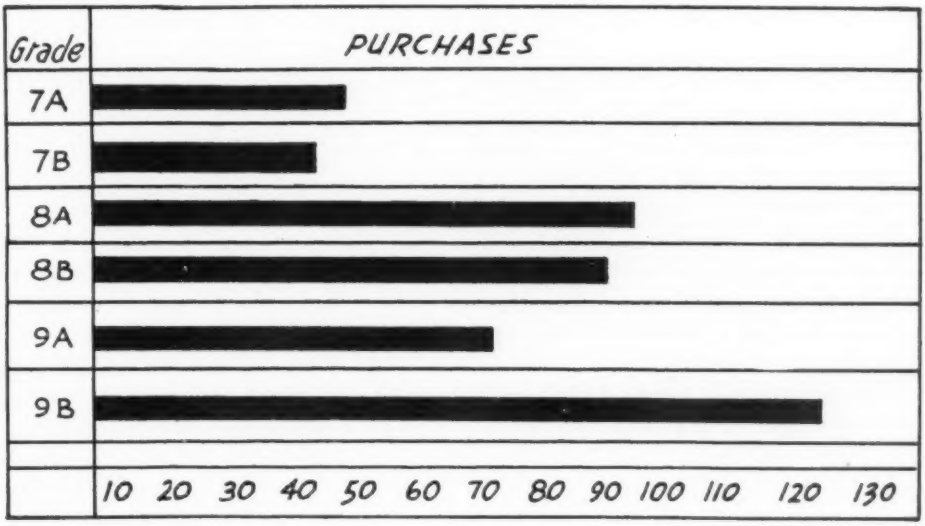
II. One family has a total income of $70 a week. Their budget for expenditures is in the second table on the preceding page.
   1. If this family used the budget rates given above, how much would they allot for each purpose listed in the table of this example?
   2. Find the difference between the amounts they are spending and the amounts recommended in the budget plan.
   3. For which purpose does it appear that they are spending:
      a. Too much? b. Too little?

III. Louise Smith budgets her Saturday earnings so that she saves 75¢ each week. In how many weeks will she be able to buy a War Bond costing $18.75?

IV. Tom Richards earns $18 a week selling newspapers and magazines. Since he is living at home, he does not have to pay for shelter, food or operating expenses. What would be a good budget plan for Tom to use? How many War Stamps should he be able to buy each week?

V. Tom's "partner" Henry earns the same weekly salary. His widowed mother has been ill and Henry gives her $10 each week to help on household expenses. What is a good budget plan for Henry to use? How many War Stamps should Henry be able to buy each week?

VI. Estimate your own weekly income from gifts, earnings, and allowance. Now set up a budget for yourself based on your expenses, family obligations, and savings. How can you increase your weekly purchase of War Stamps?



Graphs

The making of graphs requires accurate use of statistical data. The use of facts about War Savings activities in the school will impress pupils with their own progress.

I. Purchases of War Bonds and Stamps at Stuart Junior High School in one week. (Shown on graph above.)
   1. Which grade made the largest investment in War Stamps on this Stamp Day? How many dollars worth did that grade buy?
   2. Which grade bought the smallest amount?
   3. Name the grades in order of the

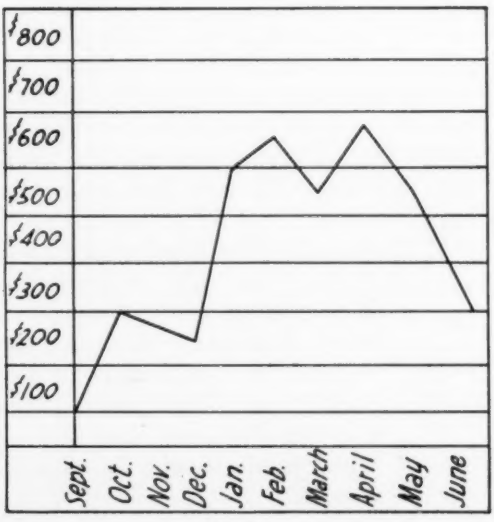
amount of their purchases, starting with the highest.

4. The 7B sales are what per cent of 7A? What per cent less than 7A?
5. The 9B sales are what per cent of 9A? What per cent greater than 9A?
6. Write the amounts invested by each grade.
7. Find the total amount invested.
8. If an Army parachute costs $65, how many could be purchased by the money invested by the Stuart students that week?
9. Prepare a bar graph showing the purchases of the different grades in your school in one week.

II. Find how much the various grades in your school invested in War Bonds and Stamps during the last month. Make a bar graph of the data.

III. Mr. Earl Winfield put 20% of his salary into War Bonds, and spent 10% for clothing, 25% for food, 20% for shelter, 5% for advancement, 15% for taxes and 5% for operating expenses. By means of a circle graph show how his salary was distributed.

IV. Purchases of War Bonds and Stamps by Grade VII in the Juneau School



1. From the broken line graph above, tell the amount of War Bonds and Stamps bought by this grade during each school month.
2. In what month were the purchases highest?
3. In what month were the purchases lowest?
4. Did your grade do as well as the 7th grade in the Juneau School?

V. Keep a record of the purchases made by the pupils in your room over a period of time such as one month. Construct a broken line graph using those data.

VI. The most important goal in a school's War Savings Program is 100% participation of the students. It is a greater achievement to have every pupil in a class buy a 10¢ War Stamp on the weekly Stamp Day than to have large sales to only a few pupils from wealthy families.

1. Keep a record of the number of pupils in your class who buy at least one War Stamp each weekly Stamp Day. Construct a broken line graph using these data.
2. Work out the percentage of participation for each week. Make a graph based on these data.
3. Using this basis of comparison, how does your class or school compare with other classes or schools in your community?

If a school can show that 90% of its pupils have participated in the War Savings Program during one month, it is entitled to fly the official white and blue Schools-at-War flag. Can your school qualify to fly this flag? Ask your school principal or local War Finance Chairman for full particulars about flying the flag.]

War Bonds

A War Bond is the written promise of the Government to repay a sum of money loaned to the Government. A Bond which

is purchased for $18.75 increases in value until it is worth $25 after ten years. This is called the maturity value. This increase in value is the "rent" or interest which the Government pays the lender for the use of the money. It is to the advantage of the owner to hold his Bonds until maturity in order to get the most for his money. Of course, the Government wants to use the loan for the full ten years in order to finance the war.

How Bonds Grow in Value

After End of Year	The Value of a $25 Bond is:	After End of Year	The Value is:
1	$18.87	6	$21.00
2	19.12	7	22.00
3	19.50	8	23.00
4	20.00	9	24.00
5	20.50	10	25.00

The table by which each of the other Bonds increases in value may be made easily as follows:

The $50 Bond is just twice the $25. Therefore, the values in the table for the $50 Bond are just twice those above.

I. What is the value at the end of 2 years of a:
   1. $25 Bond? 2. $50 Bond?
   3. $100 Bond? 4. $500 Bond?

II. 1. If a $25 Bond is purchased September 1, 1943, for $18.75, when will it mature?
   2. What will be its value when it matures?
   3. If such a Bond is purchased each month starting in September 1943, what will be the monthly income after ten years?

III. In ten years, there will be many new inventions such as radio-television sets, helicopters, etc. Name some of the many things you may wish to purchase with the money you will get when your Bonds mature.

IV. Post-war bicycles will be easier to pedal, made of magnesium or pressed wood, strong as steel and half as heavy, and may cost as little as $25. How much will you have to lend Uncle Sam now in order to have enough to buy such a bicycle ten years from now?

V. By 1953, airplane travel will have made the world a very small place. Perhaps a trip to Alaska may then be made for about $300. If you are buying War Bonds at $18.75 now, how many of the matured Bonds will be needed to make such a trip?

VI. Helicopters will sell for as little as $900 it is said. Peter's father is investing $75 per month in War Bonds. In how many months will he have enough Bonds in order to buy such a helicopter ten years hence?

Taxation

Modern warfare is expensive: A Flying Fortress costs $450,000; a heavy bombardment plane costs $300,000; a fighter plane, $150,000; light tanks cost about $45,000; heavy tanks, $145,000.

How does the Government pay for the many expenses of warfare? It can levy taxes and borrow money by means of War Bonds and Stamps. In selling War Bonds, the Government is really borrowing money on which it must pay interest. Experts agree that taxes are the cheapest way of paying for the war. Furthermore, people can pay larger taxes in wartime because many of the things one would buy in ordinary times are not manufactured during a war and because more people are earning money.

I. In his budget message of January 11, 1943, the President said that we would spend about $100 billion on the war between July, 1943, and July, 1944.
   1. Assuming that there are 130,000,000 people in the United States, how much would that amount to per person? Express the result correct to the nearest $100.
   2. To the nearest dollar, what would that amount to per person per day?
   3. The President recommends that we pay about half of the above war ex-

pense from taxes. How much will have to be raised from taxes?

II. In the year starting July 1, 1943, federal revenue may be increased from $35 billion to $51 billion yearly.
   1. How much is that increase?
   2. The increase is what per cent of $35 billion?[3]

Part Four—War Savings Problems for Grades IX to XII

Introduction

For older student questions of finance become more personal and more vital. Many of these boys and girls are working and earning their own money. Increased responsibilities in the home and increased earning power are giving them practical experience in earning, buying, saving, budgeting, and investing. Full-time employment or service in the armed forces is only a few months away. They must be prepared to handle the resultant financial problems efficiently and patriotically.

The informed citizen must recognize that his personal savings are needed by his country and our Allies. He must see that the careful handling of his own finances will contribute to the economic stability of the nation. Realistic problems worked out in the high school mathematics class will increase this understanding of the financial responsibility of the good citizen.

In addition to the problems which arise from personal earning and budgeting, from national borrowing and taxation, and from War Savings facts, many problems are suggested by current news items. For example:

1. "We lost twenty bombers." What was the approximate cost of these bombers if we assume that they were Flying Fortresses?
2. In the operations over Germany "we lost 36 bombers and 11 fighting planes." Find the money value of the lost planes.

[3] For additional "Facts and Figures for Making Problems" see Appendix on pages 373–375.

Numeration

Special instructions about large numbers is desirable.

1. Make certain that students understand the meaning of and the manner of writing millions and billions. Teach the following abbreviations
   $1 million for $1,000,000
   $2.5 billion for $2,500,000,000
2. Try to dramatize large numbers
   a. Earning $2000 a year, how long would it take to earn:
      $1 million? $1 billion?
   b. How many persons could be paid a pension of $1000 a year out of:
      $1 million? $1 billion?
   c. How many could be paid $500 a year?
   d. What does a $1000 War Savings Bond cost? How many such Bonds must be bought to provide the Government with
      $1 million? $1 billion?

Planned Saving

Probably no two individuals and no two families have identical incomes, obligations, and financial responsibilities. Some may have certain indebtedness to pay off before they can save. Others may have a number of dependents whose requirements curtail savings.

At present the great majority of families have more wage earners making more money than ever before. Although their living expenses may have risen, their luxury spending has been curtailed by rationing, travel limitations and the scarcity of consumer goods. No rule for thrift can be devised which is suitable to every person or every family. You must figure it out yourself by examining your income and necessary expenses, by searching for possible cuts in your expenses, and by investing every extra dollar in War Bonds. A patriotic citizen will recognize his responsibility to curtail his spending, plan his savings, and buy War Bonds to the limit of his ability. Again "Figure It Out Yourself!"

1. Turn back to Part Three, p. 375 to review the problems in budgeting. Work out similar problems based on typical wages which you know exist.
2. On the first of each month in 1942, Mr. Thomas bought a $25 Bond.
   a. What did each Bond cost him?
   b. What is the maturity value of each Bond?
   c. When does each Bond mature?
   d. How much will Mr. Thomas receive each month in the year when his Bonds mature, if he holds all of his Bonds until then?
3. By 1943, Mr. Thomas had secured a new job and was earning $25 a month more than he did in 1942. He and Mrs. Thomas decided that they would try to save as much as possible of these new earnings and to buy a $50 Bond each month in 1943. Answer the questions (a) to (d) of Example 2 for this new set of conditions.
4. Mr. Edwards, talking to Mr. Thomas decided that the Thomases "have something." Mr. Edwards and his wife are both making good salaries. They decide to lend the Government $150 a month.
   a. What kind of War Bonds can they get monthly with that amount of money?
   b. If they started in January, 1943, to buy the Bonds in accordance with that plan, keep it up for the next ten years, and hold all the Bonds until they mature:
      (1) When will their Bonds start to mature?
      (2) What will the maturity value be?
      (3) How long will they have Bonds maturing?
5. Helen Court in the eleventh grade works as a typist after school and on Saturday. She earns $15 a week. Her only regular expenses are $1.50 weekly for her school lunches, $1.25 a week for carfare and $.90 each week for income tax. She decides to buy one War Bond costing $18.75 every three weeks.
   a. How much will this leave her for incidental expenses?
   b. What per cent of Helen's weekly income is the amount she budgets for each item?
6. Mary Andrews, Helen's best friend, also earns $15 a week, Mary pays $8 a week to help her mother who is unable to work. She allots $2 a week for clothes, $1.50 a week for school lunches and $.90 weekly for her income tax. She is buying two 50¢ Stamps each week.
   a. How much will this leave her for incidental expenses?
   b. What per cent of Mary's weekly income is the amount she budgets for each item?
7. Mr. and Mrs. Wilson have been living on an income of $40 a week. Now with both working they are earning $100 a week. They have decided that they will not only "help to win the war" but will "make hay while the sun shines" by saving as much as they can of the extra money they are now receiving.
   a. They find that their income taxes will be about $875. How much does that leave them out of their annual income?
   b. If they use 15% of their incomes to buy Bonds:
      (1) How much will they invest in Bonds during the year?
      (2) How much will their savings bring them ten years hence if they keep all of their Bonds?
      (3) How much will they have left after paying their taxes and buying Bonds?
      (4) How much more or less is this than they lived on in pre-war days?
   c. After doing the arithmetic for part *b*, they decide that they ought to be able to save more than 15% of their income, so they figure out

the answers for the same 4 questions if they lend the Government not 15% of their income but:
(1) 20% (2) 25% (3) 30%
(4) 40% (5) 50%
Do the figuring that Mr. and Mrs. Wilson did.

d. Because they have given up their car, a vacation trip, and other luxuries, the Wilsons realize that they will not spend more during the war than they spent before. They determine to save the difference between their new wartime income and the old income plus taxes. Which per cent worked out in *c* is the best one for their War Bond allotment?

8. For the next two weeks keep a record of your expenses and receipts, including gifts, earnings and savings.
   a. Itemize these according to general classes as suggested on p. 362 of Part III.
   b. Estimate the percentage of your total income which is spent under each division.
   c. How does your record compare with the recommendations for many adults? Is your rate of saving up to the highest possible point?
   d. Now check over your expenditures. Which ones could have been cut or eliminated entirely by more careful planning and greater sacrifice on your part?
   e. How can you increase your savings next week?
   f. How can you help your family increase the family savings for War Bonds?

The Payroll Savings Plan

To assist wage earners in the systematic purchase of War Bonds the U. S. Treasury Department has sponsored the Payroll Savings Plan. By this an employee may authorize an allotment from each pay check to be paid for War Bonds. In this way the savings are allotted to War Bonds before they reach the wage earner. He is saved the trouble of going to the bank and buying his Bonds from time to time. The Bonds are delivered to him by his employer just as his pay check is delivered. By this plan he is more likely to keep to a regular schedule of saving. At the same time the Government knows how many War Bonds it can count on each month. By May, 1943 over 26.5 million Americans were buying War Bonds each month through the Payroll Savings Plan. Their savings totaled over $400 million each month.

In order to enroll in the Payroll Savings Plan, an employee must sign an authorization card giving the amount of his savings, the name and address of the Bond owner and the co-owner or beneficiary.

* * * *

PAY ROLL AUTHORIZATION for purchase of Series E War Savings Bonds

By Employee _____
(Last name) (First name) (Middle name or initial)

To Employer _____
(Company)

I hereby authorize you to save the following amount, or percentage, from my earnings every pay day, beginning with the pay-roll period ending _____, 19_____:

$_____; or 25%_____; 20%_____; 15%_____; 10%_____

* * * *

1. After four hours of school work, Tom Reardon worked in an airplane factory four hours each day and eight hours on Saturday. He earned $1 an hour.
   a. What was his weekly salary?
   b. Tom had no dependents and no living expenses. His income tax was $2.90 each week. Without much thought he signed up for 10% payroll allotment for War Bonds. Why wasn't this enough?
   c. If you were earning as much as Tom what per cent of saving would you authorize? Figure it out yourself!
2. If you are a wage earner, estimate your own expenses, earnings, and possible savings. Make out a sample budget for yourself to figure out the Payroll Savings you should authorize.

Taxation

A necessary part of planned spending is provision for income taxes. It is estimated that seven-eighths of the national income is earned by wage earners making less than $5000 a year. To assist these people in paying their share of taxes while they are earning, the Government will collect income taxes at the source. This is known as the "Pay-As-You-Go" Plan or the "Withholding Tax," since the tax will be withheld from an employee's wages and paid directly to the Federal Government. Wage earners should be able to estimate this deduction and must plan for it in making their budgets.

1. A person receiving $40 a week is in the largest group of wage earners.
   a. If that person is unmarried and does not support a dependent, his employer will withhold $6.40 from his wages each week to pay his income tax. How much of his salary remains for him?
   b. If he allots 20% of his salary for the purchase of Bonds, how much does he save weekly in this way?
   c. How much of his salary now remains to use for his ordinary expenses?
   d. How much does he pay as income tax during the year? How much does he invest in Bonds during the year?
2. If a married man earning $40 a week has one child, or other dependent, his employer will withhold $2.50 each week. For this person, answer questions b, c, d of Problem 1 above.
3. The Withholding Tax is not an additional tax. It is merely a new method of collecting the old income tax, enabling the wage earner to pay as he earns instead of having to save for quarterly income tax payments.
   a. Suppose you were offered a part-time job paying $20 a week. Estimate your expenses, tax deduction, and Payroll Savings authorization. (In estimating your tax deductions your employer will probably use the deduction chart authorizing that $1.90 be deducted weekly from the wages of a single person without dependents and earning $20 a week.)

Price Increases

It is estimated that in 1943 American citizens will have $125 billion after they have paid their taxes. Actually there will be only $80 billion worth of consumer goods for these citizens to buy with their $125 billion. The difference between the available money and the available goods ($125 B − $80 B = $45 B) is called the "inflationary gap." If these wage earners should go on a "buying spree" and decide to buy goods at any price, they would bid against each other for scarce consumer goods. That would cause an abnormal rise in prices so that purchasers would be paying for more than an article's real worth. On the other hand, if they save a large portion of this excess $45 billion, their money will be helping to win the war instead of competing for scarce goods at home.

1. The following table shows the amount

of money that consumers have had in recent years over and above the dollar value of all the "consumer goods and services" available:

In Year	1941	1942	1943
Amount	$9 B	$25 B	$45 B

a. Represent these amounts by a horizontal bar graph.
b. 1941 was the last pre-war year. How does the amount for 1943 compare with that for 1941?

2. An example of receipt of increased income. Mr. Jones received $45 a week before the war; now he earns $65 and his wife $30 a week.
   a. During the year how much more can Mr. and Mrs. Jones spend than before the war?
   b. Their car is old. Why can't they buy a new one? Name some other ways in which they might like to spend their increased income but cannot.
   c. Mr. Jones "likes a good steak." How may he be tempted to interfere with rationing and the ceiling prices on steak and other scarce items? Why is this unpatriotic and unwise?
   d. Out of their total annual income, Mr. and Mrs. Jones set aside 20% for purchase of War Bonds, and have about $1,000 withheld as income taxes. How much does that leave them out of their total income for the year?
   e. What per cent of the pre-war income of Mr. Jones is the result of *a*?

4. In pre-war years Mr. Smith earned $175 a month; now he is getting $200 a month.
   a. Find the annual income of Mr. Smith in pre-war years.
   b. In pre-war years, Mr. Smith paid about $36 as income tax and saved 10% of his income. How much was left after paying his income tax and setting aside his savings?
   c. What is the annual income of Mr. Smith now?
   d. His present income tax is $230 and he saves 20% for War Bonds. How much is left annually for all other purposes now?

Graphs

Review pp. 363 to 364 of Part Three for use of statistical data in the making of graphs. Similar problems can be developed using more advanced information and more complicated procedures. In any case, students will gain a deeper sense of their own responsibilities to earn and save. Through the use of facts and figures related to the current war finance program, students will gain a wider sense of personal responsibility at the same time they are improving their mathematical skills.[4]

Part Five—Tested War Savings Projects

Classroom Study

One of the most successful projects is the classroom study of War Savings investments, budgeting, and ways to save. Pupils like to work out their own problems in terms of War Stamps and the military equipment which such savings will buy.

Let the class divide into teams to prepare problems. Then have Team A try to solve the problems prepared by Team B. If there is a faulty problem which cannot be solved because of conflicting information, the team which made the problem should be penalized. Lively competition and sound learning situations have resulted from this sort of classroom study.

Quiz Program

Perhaps your students would profit from a quiz program in the school assembly or over the radio. The questions could be based on information which every citizen should have about War Stamps and

[4] For additional "Facts and Figures for Making Problems" see Appendix on pages 373–375.

Bonds. Simple problems would be effective tests for your local "Quiz Kids."

Why not encourage your students to challenge another class in such a battle of the wits? Or better still, have the School War Savings Committee challenge the corresponding committee in your community. If the student council is ready to challenge the city council on its knowledge of War Savings facts and figures, you have already the makings of a good program with popular appeal.

Selecting Sales Goals

When a class or school selects certain War Savings goals in advance, there is greater incentive for students to invest regularly and to the utmost. In one state a quota was set for each school in terms of selected pieces of military equipment. In order to determine the quota, previous sales were totaled and found to average $1.25 per pupil per month. New goals were set at $1.50 per pupil. However, these quotas were worked out in terms of military equipment to be paid for by the school and were not announced in the dollar and cents value. Hence, a school with 2000 pupils was given a monthly goal of "paying" for a Grasshopper plane which costs $3000. This was selected since the $1.50 savings of each of 2000 pupils would total $3000, the price of the plane. War Savings chairman reported the new goals were oversubscribed in almost every case.

Mathematics classes could contribute to the effectiveness of their school War Savings Campaign by working out such goals for every home room group.

Keeping Records, Graphs, Charts

In order to keep up the rate of progress, pupils like to see how far they have to go to reach a goal. Schools are making an increasing use of thermometer charts of various types, based on the amount of sales, and the per cent of participation. Usually there is a progress chart showing the amount of sales week by week. Frequently mathematics classes have joined the art department in preparing such a chart for the main entrance to the school, the auditorium stage, the gymnasium or some other prominent place. Art students design the chart or graph and actually build or paint it. Mathematics students keep the records and are responsible for weekly additions to show the progress which has been made.

Triple-Threat Jeep Campaign

Between the opening of school and Pearl Harbor on December 7, schools are urged to participate in the Triple-Threat Jeep Campaign to finance the purchase of various types of jeeps through War Stamps and Bonds. The regular land-going variety, or jumping jeep ($1165) has two cousins —the swimming jeep or "quack" ($2090), and the flying jeep or Grasshopper plane ($3000). These three provide the triple-threat on land, on sea and in the air. Schools which finance one or more of the jeeps and report the sales to their State War Finance Chairman, will be awarded a Treasury Citation. For full details see "Handbook for School Administrators, 1944 Issue," to be sent to every school principal and superintendent.

Buy a Bomber Campaign

Larger schools, or groups of schools can actually name a plane by successfully completing a campaign to pay for one of the following through purchases of War Bonds: Pursuit Plane, $75,000; C-3 Ambulance Plane, $110,000; Medium Bomber, $175,000; Heavy Bomber, $300,000. Before any such campaign is scheduled, however, plans should be submitted for approval to your State War Finance Committee. School representatives will not be permitted to be present at the christening, but a photograph of the plane with the name painted on the nose, will be made by the Signal Corps, and forwarded to the school. This takes about 12 weeks.

Adopt a Soldier

Many schools have set their War Savings goals in terms of the cost of equipping

and maintaining a soldier. His needs keep the class to a regular savings program.

Letter Home

When students have studied about War Savings and their own part in the program of national finance, they are better equipped to assist their parents in making and keeping the family budget. After full discussion, have each of your students write a letter home explaining the importance of regular savings, how family savings may be raised, and the necessity for increasing War Savings purchases. In some schools students have actually worked out family budgets to submit with the letters. This is an effective way of reaching parents, as well as a good method to employ in promoting better home and school relations.

Community Drives

War Loan Drives will be made throughout the country at regular intervals. During these drives, schools may provide excellent publicity through such devices as "The Letter Home," the community progress chart, the radio quiz program, a Town Meeting on War Finance, or through newspaper stories based on War Savings facts and figures.

All of these school projects reaching into the community are most effective when accomplished through the cooperative efforts of various departments. For example, mathematics classes can provide interesting facts and graphs for speech students who will go on the air or address civic organizations. Art classes can dress up the usual statistical chart of the school or community. Home economics students can assist in working out budgets and publicizing them in their classes and homes.

The need for faculty and student leadership in the entire War Savings program is unending. Your resourcefulness and ingenuity as well as your recognized influence in the community can provide one of the most effective assurances of success in this phase of the Battle of the Home Front. Will you accept the challenge?

WAR SAVINGS MATERIALS FOR TEACHERS

**Schools at War, A War Savings News Bulletin for Teachers*, distributed quarterly to every teacher. Contains suggestions for classroom study, news of school War Savings projects, and school posters, which are loose leaf for bulletin board use.

**Help Send Them What It Takes To Win*, 4-color poster depicting a typical debarkation scene with illustrations of many pieces of military equipment with prices. Additional facts and figures as well as arithmetic problems are on the back.

**Questions and Answers about United States War Savings Bonds and Stamps*, printed leaflet answering 98 questions about War Bonds of Series E.

**$100 Billion for War*, prepared by Mabel Newcomer, Professor of Economics, Vassar College. A concise and factual study of America's wartime economics.

Paying for the War, A Resource Unit for Social Studies Teachers, gives the costs of previous wars and how they were financed, and the current Government program of taxation and borrowing. Published by the National Council for the Social Studies, 1201 Sixteenth Street, N.W., Washington, D. C. 30¢

How To Win On the Home Front, Public Affairs Pamphlet No. 72, contains practical information about family budgeting and simple ways to save. Public Offairs Committee Inc. 30 Rockefeller Plaza, New York City. 10¢

**The Teacher of English and the War Savings Program*, including two units for high school English classes.

**Figure It Out Yourself*, Payroll Savings leaflet explaining the need for regular War Savings investments and how to sign up for payroll allotment.

**Handbook for School Administrators*, 1944 Issue. Guide to a good school War Savings program with suggestions for sales, classroom study, community activities, payroll savings, and available materials.

* Available free on request to Education Section, War Finance Division, U.S. Treasury Department, Washington 25, D.C.

Appendix—Facts and Figures for Making Problems

I. Cost of Selected Supplies and Equipment

A. Clothing for an enlisted man in the army $108.59

1 woolen coat	$11.53	2 twill jackets	$ 4.32
2 woolen trousers	12.38	2 twill trousers	4.16
1 overcoat	15.50	1 twill cap	.48
2 pair shoes	8.62	2 mohair neckties	.44
3 khaki shirts	6.96	1 web waist belt	.23
1 garrison cap	1.03	1 pair woolen gloves	.99
1 cotton cap	.63	4 handkerchiefs	.20
3 pair khaki trousers	7.80	1 steel helmet with lining	3.66
2 woolen shirts	9.02	3 pair tan socks	.51
4 cotton undershirts	.88	3 pair woolen socks	.96
2 woolen undershirts	2.98	2 flannel shirts	9.02
1 field jacket	6.82	1 raincoat	5.35

B. Barrack and personal equipment for one man ..... $ 41.99

1 mosquito net and rod	$ 3.16	2 towels	$ .32
2 woolen blankets	14.26	2 barrack bags	1.64
1 comforter	3.05	1 field bag	1.30
1 mattress	6.47	1 cartridge belt	.83
1 pillow	.80	2 pistol belts	.62
2 pillow cases	.62	1 canteen cup and cover	1.59
1 folding chair	.80	1 haversack	1.53
1 folding cot	3.40	1 first aid pouch	.27
1 set toilet articles	.87	1 mess kit	.93

C. Approximate cost of clothing and equipment for:

Enlisted man in the army	$164
Enlisted man in the Marines	200
Enlisted man in the Navy	135
A member of the WACS	127
A member of the WAVES	189
A member of the SPARS	189
An Army Nurse	250
A Navy Nurse	250

D. Army equipment:

Bayonet		$ 5.00
Bomb—(100 lb.)		41.63
Bomb—(4000 lb.)		872.42
Garand rifle bullet		.055
Cars: Jeep (¼ ton)		1,165.00
Amphibian Jeep or "Quack"		2,090.00
Reconnaissance		1,580.00
Scout		7,500.00
Half-track		10,500.00
Dog of War—collar		.60
leash		.55
food	a day	.30
Food—for man in United States	a day	.60
for man abroad	a day	.85
First aid kit (Aircraft)		4.89
Guns: Anti-tank (37 mm.)		3,400.00
Anti-tank (75 mm.)		10,000.00
Anti-aircraft (Bofors 40 mm.)		25,000.00
Anti-aircraft (90 mm.)		43,800.00
Howitzer (75 mm.)		11,350.00
Howitzer (105 mm.)		13,400.00
Howitzer (155 mm.)		32,700.00
Browning machine gun (.50 cal.)		740.00
Machine gun (.30 cal.)		275.00
Mortar (60 mm.)		315.00
Mortar (81 mm.)		650.00

Submachine gun	55.00
Motorcycle	400.00
Parachute	65.00
Pistol (.45 automatic)	50.00
Airplanes: Flying Jeep or Grasshopper Plane	3,000.00
Fighter	150,000.00
Primary trainer	25,000.00

Larger schools, or groups of schools can name a plane by purchasing enough War Bonds to finance one of these:

Pursuit ship	$ 75,000
C-3 Ambulance Plane	$110,000
Medium Bomber	$175,000
Heavy Bomber	$300,000

Airplane engine	$12,000 to 14,000.00
Rifle	85.00
Searchlight (anti-aircraft)	27,500.00
Sound locator	5,000.00
Stretcher (field hospital)	17.20
Tanks: Light unarmored	45,000.00
Heavy (60 tons)	145,000.00
Trucks: 2½ tons	2,210.00
10 tons	8,360.00
Walkie-talkie	200.00

E. Navy equipment:

Aircraft torpedo	$ 11,000.00
Ships: Battleship (35,000 tons)	85,000,000.00
Aircraft-carrier	71,000,000.00
10,000 ton cruiser	35,000,000.00
Heavy cruiser	77,000,000.00
Destroyer (heavy)	12,000,000.00
Submarine	7,000,000.00
Minesweeper	3,500,000.00
Boats: Aircraft rescue (45 ft.)	20,000.00
Collapsible (8 persons)	1,500.00
PT	145,000.00
Motor torpedo	540,000.00
Landing barge (ocean-going)	2,000,000.00
Landing craft (36 ft.)	12,500.00
Life-Float (with oars and rations for 7)	350.00
Submarine chaser	500,000.00
Airplanes: Primary trainer	13,000.00
Advanced trainer (2-engine)	70,000.00
Fighter	70,000.00
Dive bomber (2000 hp.)	110,000.00
Patrol bomber (2-engine)	250,000.00
Patrol bomber (4-engine)	500,000.00
Transport (4-engine)	400,000.00
Clothing and equipment for one enlisted man	135.00
Cost of firing one broadside from a battleship	13,500.00
Compass (4 inch)	31.50
Depth charge (300 lb.)	105.00
Life-belt (self-inflating)	4.75

II. War Savings Bonds (See *Questions and Answers about United States War Savings Bonds and Stamps*).

A. Series E War Savings Bonds may be purchased in 5 denominations as follows:

Bond Purchase Price	Maturity Value 10 years later
$ 18.75	$ 25.00
37.50	50.00
75.00	100.00
375.00	500.00
750.00	1000.00

B. If held to maturity (10 years) Series E Bonds yield at the annual rate of 2.9 per cent interest compounded semiannually.

C. Each Bond is registered at the Treasury Department. It can be registered only in the names of individuals in their own right, in one of the following forms:
   1. The name of the individual, or
   2. The names of two individuals as coowners, or
   3. The name of one individual as owner and one other individual as beneficiary.

D. War Bonds cannot be sold to other individuals or given away.

E. They may be redeemed after 60 days from date of issue if the owner feels this is absolutely necessary. So far 96% of the Bond purchasers are still holding their Bonds.

F. War Bonds are owned by over 50 million Americans.

III. Payroll Savings Plan

Persons using it and per cent of wages allotted by them.

Month	No. Persons	Per cent	Month	No. Persons	Per cent
Mar. '42	9.6 M	4.9	Oct. '42	22.6 M	7.8
Apr. '42	11.6 M	4.9	Nov. '42	23.7 M	8.3
May '42	13.9 M	5.3	Dec. '42	24.5 M	8.5
June '42	16.0 M	5.8	Jan. '43	24.9 M	8.7
July '42	17.8 M	6.5	Feb. '43	25.5 M	8.7
Aug. '42	19.4 M	7.1	Mar. '43	26.2 M	8.7
Sept. '42	21.0 M	7.5	Apr. '43	26.8 M	9.1

Note. Use for graph to show growth of Payroll Savings Plan

IV. Suggested Plan for Making a Budget

A. List for the whole family for an average month:

Rent	$
Food	
Clothing	
Taxes	
Carfare	
Insurance	
Medical care	
Amusements	
Heat, light	
Education	
Charity	
House help	
Miscellaneous	
Total	$

List also the total income of all members of the family:

Income _____

Expenses _____

_____

Savings _____

B. In many families the savings should be from 15% to 25% of the total income, depending upon the number of wage earners and the previous indebtedness of the family. Total war demands sacrifices on the homefront as well as on the battle front.

C. A good budget plan is given on page 362.

◆ THE ART OF TEACHING ◆

Development of Interest and Skill in the Handling of Trinomial Perfect Squares

By HAROLD E. BOWIE
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AN ABILITY to recognize and form trinomial perfect squares is very useful to the student of mathematics. A good time to develop skill in this work is in connection with the solution of quadratic equations, especially in the case where completion of the square is used.

Pupils commonly like to solve quadratic equations when they can use factoring. They do not object to obtaining the results by completing the square until after they know about the formula. They find this way more difficult but feel it necessary when they cannot factor.

After the formula has been derived the bright pupil raises a natural objection, not always vocal of course, to any further use of the completing the square idea. He may even feel that any extended practice in the first place was unnecessary and that he was given work just to keep him busy.

The teacher can set the mind of the student at rest on this point, provide sufficient practice in recognizing and forming perfect trinomial squares, and use the topic to interest him in further work in mathematics.

In order to do this one must have given some thought to the importance of the idea. Some of the applications of the method of completing the square as ordinarily used in solving quadratic equations are as follows:

1. In algebra.

Example: For what values of K will the roots of the equation $X^2+KX+K=0$ be real and different?

Solution: $K^2-4K>0$

$K^2-4K+4>4$

$(K-2)^2>4$

$K-2>2$ or $K-2<-2$

$K>4$ or $K<0$.

2. In analytic geometry to throw equations into the standard forms.

Example: $2x^2+3y^2-4x+5y-6=0$

$2(x^2-2x+1)$

$+3(y^2+5y/3+25/36)$

$-6-2-25/12=0$

$2(x-1)^2+3(y+\frac{5}{6})^2=121/12$

$$\frac{(x-1)^2}{121/24}+\frac{(y+\frac{5}{6})^2}{121/36}=1.$$

This is an ellipse with center at $(1, -\frac{5}{6})$ and with axes parallel to the coordinate axes. Similar exercises involving circles, parabolas, and hyperbolas are common.

3. In the calculus to change integrands into forms to which the formulas of integration may be applied.

Example: $$\int \frac{dx}{x^2-3x+7}$$

$$=\int \frac{dx}{x^2-3x+9/4+19/4}$$

$$=\int \frac{dx}{(x-3/2)^2+19/4}$$

The integration may now be carried out by using the formula

$$\int \frac{dv}{v^2+a^2} = \frac{1}{a} \arctan \frac{v}{a} + c.$$

Other integration formulas involving arc sin and logarithms in the results are common. They often require a perfect square.

In trigonometry we find a slightly different use for the ability to form perfect trinomial squares in the development of the half angle formulas.

Example: $2 \sin^2 \frac{1}{2}A$

$$= 1 - \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{2bc-b^2-c^2+a^2}{2bc}$$

$$= \frac{a^2-(b^2-2bc+c^2)}{2bc}$$

$$= \frac{(a+b-c)(a-b+c)}{2bc}$$

Let $a+b+c=2s$

Then $a-b+c=2(s-b)$

$a+b-c=2(s-c)$

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Heron's formula for the area of a triangle $K=\sqrt{s(s-a)(s-b)(s-c)}$ as derived in plane geometry or trigonometry involves the same idea.

In work involving exponential functions it is often necessary to form a perfect square.

Example: Express $e^{2x}-2+e^{-2x}$ as the difference of two squares.

Solution: $e^{2x}-2+e^{-2x}$

$$= e^{2x}+2+e^{-2x}-4$$

$$= (e^x+e^{-x})^2-4.$$

These are only a few of the many instances where trinomial perfect squares are involved. We have given enough examples to show the importance of giving the topic considerable attention. It is not my thought that pupils should be given work in analytic geometry and calculus at the time they study quadratic equations. However, the teacher can interest them greatly by using some time to mention the subjects where the method is used and by going far enough with a few examples to create an interest.

In addition to work involving quadratic equations, practice with exercises involving a number of variations of the idea should be used. The following examples may suggest others.

1. Express the polynomial x^2-3x+5 as a trinomial perfect square plus or minus a constant.

2. Write the expression $2a^2-2ab+b^2$ as a trinomial perfect square plus a monomial.

3. Factor a^4+4b^4.

Spending a little more time than usual on this topic will give the pupil skill which will be of much use to him later.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
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THE MATHEMATICS TEACHER

Volume XXXVI | DECEMBER · 1943 | Number 8

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OFFICIAL JOURNAL PUBLISHED BY THE

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

MENASHA, [illegible], NEW YORK, N.Y.

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

A MERRY CHRISTMAS TO ALL OF OUR READERS!

URGENT PUBLIC NOTICE!